

## The Association of Mathematics Teacher Educators <br> http://www.amte.net

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The Board of Directors and the Membership Committee are in the process of reviewing and revising the AMTE Goals. AMTE's goals were developed approximately 15 years ago, when the organization was founded, and included 12 separate goals. While reviewing our current membership brochure, the Membership Committee suggested revising the goals to more concisely describe AMTE's work. The Membership Committee submitted a proposed revision of the goals to the Board of Directors, who reviewed it and further modified it at our March Board meeting. These new goals are listed below, along with the AMTE Mission Statement, which has not changed.
AMTE members are invited to suggest modifications of these goals. Please submit them to AMTE's Executive Director, Nadine Bezuk, at nbezuk@mail.sdsu.edu no later than August 15. Suggestions will be forwarded to the Constitution and Bylaws Committee for consideration.
This amendment will be discussed at the Annual Business Meeting at the 2008 Annual Conference. If this amendment receives a majority vote of the members present at that meeting, the amendment will be submitted to the members for ratification.
AMTE's mission is to promote the improvement of mathematics teacher education in all its aspects. AMTE members are persons working in institutions of higher education, whether in education, mathematics, or other related departments, as well as in K-12 and other settings.

## The proposed goals of AMTE are to:

1. promote ongoing professional growth of mathematics teacher educators;
2. promote communication and collaboration among those involved in mathematics teacher education;
3. promote effective mathematics teacher education programs and practices;
4. promote recognition of the ever-increasing impact of technology on mathematics teacher education;
5. promote research and other scholarly endeavors related to mathematics teacher education;
6. advocate for effective policies and practices related to mathematics teacher education at all levels; and
7. advocate for equitable practices, including to support and increase the diversity of mathematics teachers and mathematics teacher educators.
Connections is published three times a year: fall, spring, and summer. The Editorial Board will consider a wide variety of types of submissions. Regular features include essays addressing each issue's Theory and Practice question, reviews of resources for mathematics teacher educators, and news articles related to mathematics teacher education. Each submission is reviewed by the editorial board for relevance to the AMTE membership and for quality of work. Please direct all comments, questions, or submissions to the editor at lstalling@kennesaw.edu or 770-420-4477.

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## 2008 AMTE Annual Conference News

Make your plans now to attend the 2008 AMTE Annual Conference in Tulsa, Oklahoma on January 2426, 2008. Based on lots of feedback from past conference attendees, this year's conference will have three new features:

- Pre-conference sessions will be held Thursday morning.
- Regular conference sessions will start after lunch on Thursday.
- The conference will end by mid-afternoon on Saturday.

The opening general session will begin at 7:00 p.m. on Thursday; it will be followed at 8:30 p.m. by a light reception. Dinner is on your own on Thursday and there are several restaurants within walking distance. As usual, all meals on Friday and through lunch on Saturday are included in your registration fee.

The conference site is the Renaissance Tulsa Hotel \& Convention Center. The hotel room rate is $\$ 135$ for a single or double room. The deadline for reservations is December 7, 2007 or when the room block is full. If the room block is filled prior to the deadline, the hotel will accept reservations at the hotel's prevailing rate and only on a space-available basis. More information on registration and details about hotel reservations and conference activities can be found on the AMTE web site. We hope to see you in Tulsa in January!

## President's Column (Continued from page 1.)

has completed extensive revisions of our constitution to incorporate our expanded initiatives and realigned goals.

- Maggie Niess continues to lead the Technology Committee. This group oversees the NTLI award at each AMTE conference. This committee also just prepared, piloted, and delivered a web site survey and developed AMTE's second Position Statement.
- Susan Friel and Peg Smith have led a Task Force on Teaching Resources. This work has led to AMTE fourth monograph, which is now in press.
- Monographs- Yes, we are starting on Monograph V - have been masterfully overseen by Denisse Thompson as the Series Editor. Thankfully, Marilyn Struchens has agreed to become the second Series Editor, overseeing the next three monographs.
- Another major long-term commitment is Susan Gay's position of Conference Coordinator. This year, Dale Oliver also provided significant support.
- Each year, the Annual Program Committee is a very busy group. Connie Schrock is the chair for 2008 Conference. Sandi Cooper chaired this year's very successful AMTE Conference Program and Stephen Pape will chair in 2009.
Leading and working in these groups is hard work! And, because it may not be as urgent as some other things on our plates, it requires commitment by chairs and members to ensure that the work is getting done. At the AMTE Annual Meeting, the committee chairs met and discussed ways to support their work. As a result, we are making some changes that are important for you to know.

One of the changes is that I will be making AMTE appointments for 2008-2010 terms in Fall 2007 (previously we have waited until after the AMTE conference). This change is so that the new members can participate in the committee meetings that often occur at the AMTE conference. The Volunteer Form for 2008 will soon be posted online. Please consider how you might best contribute to the work of AMTE (and encourage your colleagues to do so!).

Starting in 2008, we will also be sharing (through this newsletter) the work of committees in more detail as a way of better informing the membership and receiving more input from members.
Bill Bush, AMTE's first winner of the Excellence in Service to Mathematics Teacher Education Award (2007), states in an essay (p. 6) taken from the address he gave at the Annual Conference that one must make choices about the service one pursues. Pick AMTE. Service on AMTE committees is a great opportunity to meet new colleagues, work strategically on problems that can inform your local work and learn more about the organization. Service to AMTE may lead to other opportunities to serve mathematics teacher education.

Finally, thanks to the committee chairs (former and current) for their work throughout the year. It isn't easy to coordinate conference calls, prompt members to respond, and to move forward when meetings are via e-mail and telephone. Your efforts make a difference.

## Notice the

 change to the annual conference schedule: Sessions begin after lunch on Thursday, January 24, 2008.
## Nominations Sought for Positions on the AMTE Board of Directors

The AMTE Nominations and Elections Committee is seeking nominations for candidates for the positions of President, Secretary, and Member-atLarge.

The AMTE Nominations and Elections Committee is seeking nominations for candidates for the positions of President, Secretary, and Member-at-Large. Please review the job descriptions below. To nominate a candidate, send a completed nomination form to the chair of the nominations committee, Peg Smith at pegs@pitt.edu, or via snail mail to Peg Smith, 5515 W.W. Posvar Hall, University of Pittsburgh, Pittsburgh, PA 15260. The nomination form can be found at the AMTE web site http://www.amte.net. Be sure to indicate the position (president, secretary, member-atlarge) for which you are nominating the candidate. Nominations of colleagues and self-nominations are permitted. Before selecting any potential candidate to run in the fall election, the Nominations Committee will verify his/her willingness to serve. All candidates must be members in good standing of the organization.

After reviewing all of the nominations submitted by the July 31, 2007 deadline, the Nominations and Election Committee will formulate an election slate, taking into consideration both professional qualifications and diversity (e.g., years of experience, racial or ethnic background, recognized or demonstrated leadership skills).

## Send completed nomination forms to: Peg Smith (pegs@pitt.edu) Deadline: July 31, 2007

## Position Responsibilities

All members for the board of directors have a commitment to attend two (2) board meetings per year - one held prior to the NCTM Annual Meeting and the other at the AMTE Annual Conference (usually in January). The term of each office is three years, except for the president. The president of AMTE serves on the board for one year as president-elect, two years as president, and one year as immediate past-president.

## President

The President shall ensure that the affairs of AMTE are conducted in accordance with the Constitution, Bylaws, and policies of AMTE; shall be the presiding officer at the annual business meeting, Board of Directors meetings and any special meetings; shall call special meetings as provided for in the Constitution; shall coordinate the activities of standing committees; and shall provide leadership for the attainment of the goals of AMTE.

## Secretary

The Secretary shall record and maintain a file of the minutes of official meetings of the Association and its Board of Directors and shall be responsible for the correspondence of the Association. The secretary is a voting member of the board and an active participant in all board discussions.

## Member-at-Large

The Member-at-Large shall assume those responsibilities determined by the President. A major duty of the Member-at-Large is to support and advise the president of AMTE. The Member-at-Large serves as a Board liaison to one or more committees or task forces. The Member-at-Large is responsible for reporting on the progress of his/her assigned committee(s) or task force(s) at Board meetings.


AMTE Connections Summer 2007

# Service: The Very Purpose of Our Work as Mathematics Educators 

## William Bush, University of Louisville

## Congratulations

 to Dr. William(Bill) Bush, AMTE's first winner of the Excellence in

Service to Mathematics Teacher Education Award!

Summer 2007
have the balanced vita necessary for tenure and promotion. In preparing for my AMTE presentation, it suddenly dawned on me-it's all really service!

Teaching is clearly a
 service enterprise. Mathematics teacher educators serve preservice and practicing teachers by: (1) helping them grow intellectually, psychologically, and emotionally; (2) establishing mentoring relationships with them; (3) building learning communities and collaborative networks with them; and (4) shaping them into leaders in the field. All of these activities provide service not only to the many teachers touched through our work but ultimately to the students they teach.

Research is also a service enterprise. Mathematics education researchers serve the education community by: (1) posing pertinent questions and identifying critical problems in mathematics education; (2) seeking answers to questions about the teaching, learning, and the culture of mathematics education; (3) solving problems that pose as barriers to quality mathematics teaching and learning; and (4) helping mathematics teachers and others do their work effectively. And, there are plenty of challenges, questions, and problems to address.

Because all our work as mathematics educators is service (even when it is not specified as such), it is critical that we be efficient and effective about our work. All of us, and especially those new in the profession, need to be thoughtful and deliberate about how and when we serve. Effective and efficient service requires us to "look" at ourselves and the world carefully (and perhaps differently). In the columns that follow, I will use some of my experiences to illustrate ways of "looking" at ourselves and our work to help make decisions about serving others.

Looking Inward. Oprah Winfrey captures this sense of introspection: "I've come to believe that each of us has a personal calling that's as unique as a fingerprint - and that the best way to succeed is to discover what you love and then find a way to offer it to others in the form of service, working hard, and also allowing the energy of the universe to lead you." When I looked inward early in my career, I knew my passion was helping and supporting
teachers, especially preservice teachers grow into accomplished professionals. My favorite times have been working closely in classrooms with preservice and practicing teachers. In fact, I now hold a job that aligns with my passion and expertise-Director of the University of Louisville Center for Research on Mathematics and Science Teacher Development. Like all journeymen on life's road, I have plenty of non-examples of serving outside my passion and expertise. For example in 1990, I agreed to work with our statewide public television network to promote the NCTM Standards. While I had sufficient knowledge of the standards, I had little knowledge or passion for producing promotional materials. As a result, the time effort I invested in the project distracted from other service I could have done, denied someone with the passion and expertise for this work an opportunity to contribute, and probably annoyed others who worked on the project.

To be effective in service, we must be aware of our personal passions, interests, strengths, weaknesses, and personal styles. We must be introspective enough to focus our service efforts on those activities that suit us and that maximize our potential impact. In many cases, it is fine, and sometimes advantageous, to reject service opportunities that simply do not fit our expertise or passions.

We also must be aware of the scope of our goals; that is, at what level (local, regional, state, national, or international) should we focus our service? Although I have been involved in a number of national activities and initiatives, my impact choice has been at the state level. Born and raised in Kentucky, I have a passion to focus in Kentucky because of those teachers and faculty whose service benefited me. The decision about impact goals may change over time with circumstances. Of course, young faculty members generally focus on local initiatives, then expand to state and national impact with experience and opportunity. In any case, it is always important to identify where our passion and expertise lie because there is so much to do and so little time to do it.
Looking Outward. In 1989, the unveiling of the NCTM Standards provided an opportunity for a group of mathematics educators and mathematicians in Kentucky to look outward. This group met on several occasions to create a strategy for the statewide adoption of the Standards. In the conversations that ensued, we realized that Kentucky did not have the leadership capacity to promote or implement the Standards in a significant way. This discussion led to the development of two NSF projects-the Kentucky K-4 Mathematics Specialist Program and the Kentucky Middle Grades

Mathematics Teacher Network-that over a sevenyear span helped develop about 100 mathematics leaders who worked with nearly 2000 teachers. As a result, many of those 100 individuals assumed formal leadership roles in local schools, districts, colleges, and the Kentucky Department of Education in subsequent years.

We cannot serve effectively unless we see the needs of colleagues, students, teachers, and schools. Solving problems in mathematics education is like solving problems in mathematics in that we must fully understand the problem before we attempt a solution. We have to have insight into the mathematical, learning, teaching, and personal problems of others, and to do that we must be good listeners. We must hear the real needs of students, teachers, administrators, and parents.

Effective servers must also understand the world. Knowing the contexts, conditions, limitations, and forces in the world in which we live and work gives us insight into strategies and solutions to resolve challenges and solve problems. It is critical that we know research. We must be effective producers and consumers of research. We should be aware of programs and practices proven effective by research, as well as programs and practices with little or no empirical support. We are the logical experts to assist teachers, administrators, and parents in synthesizing and interpreting research about our field.

Looking Around. The NSF-funded Appalachian Collaborative Center for Learning, Assessment and Instruction in Mathematics (ACCLAIM) serves as an excellent example of the power of collaboration. This project was effective in constructing a nontraditional mathematics education doctoral program across five universities, building leadership opportunities for mathematics teachers across four states, and blending mathematics education and rural education research with the collaborative effort of mathematics educators, mathematicians, and rural educators. Service is much more effective when it involves collaborative efforts. Working collaboratively enhances the quality of service by increasing intellectual power, building upon individual strengths, diversifying perspectives, and tapping into the passions of others. As we consider service activities, identifying collaborators reduces the work while increasing the impact.

Looking Smart. Early in my career, I conducted workshops for school districts simply because they asked me. In some cases, the workshops were absolute disasters because what I had to offer was not what the teachers wanted to know. Neither the districts nor I had checked with the teachers to
(Continued on page 9.)

## To be effective

 in service, we must be aware of our personal passions, interests, strengths, weaknesses, and personal styles.AMTE Connections Summer 2007

# MathNerds Online Q \& R Service Connects <br> Pre-service Teachers and School District Students 

Laurie Cavey, James Madison University

In this issue of Connections, several new columns debut. In the Project News Column, members will be asked to submit descriptions of projects or centers. The purpose of the column is to inform members about projects before they get to presentation and publication stages. Submissions of 1000-1200 words should be directed to column editor, Laurie Cavey (caveylo@jmu.edu)

For more than ten years, the non-profit MathNerds (Dawkins, De Angelis, Mahavier, Stenger, 2002) has provided a free, web-based, question-and-response service supplying guidance in mathematics to students around the world. Over the past three years, the site has responded to about 1,500 questions per month with an average response time of approximately 16 hours. Through personal profiles, volunteers control the number of questions they receive and the categories ( $\mathrm{K}-12$ through graduate) in which they receive questions. Clients submit questions online that are routed randomly to the volunteers who have agreed to respond to questions in that category and who have not met their weekly quota. MathNerds has a strong commitment to inquiry-based education, teaching people to teach themselves and striving to avoid contributing to the abuse of the internet by doing homework, take-home tests, or school-related projects. Volunteers are committed to providing individual guidance, references, and hints - not answers per se.

Recently, MathNerds has developed Mentoring Networks to connect school districts to local universities. Following MathNerds' inquiry-based question-and-response model, a system was developed where middle grades and high school students submit questions through the web site that are routed directly to prospective teachers taking a
methods class. Each network aims to connect a local school teacher(s) with a university mathematician and mathematics educator, who each carefully monitor the questions and responses. Analysis of the questions and responses of each pilot program is ongoing and has initially prompted a closer examination of what prospective teachers are learning through participation in the online dialogues (Cavey, Mahavier, Parker \& White, in press). New programming for the fall 2007 Mentoring Networks will enable pre-service teachers to work collaboratively to develop a response to a given question. New programming will also make it possible for the university and school district partners to review responses before being routed back to the school district student. Universities currently participating in the Mentoring Networks include: James Madison University, Lamar University, and Texas State University.

Additional partners are always welcome! If you are interested in setting up a pilot network at your university or school district, please contact W. Ted Mahavier (wtm@mathnerds.com). To hear more about the mathematics educator's perspective, contact Laurie Cavey (caveylo@jmu.edu). For additional information see our web site: http:// www.mathnerds.com/mathnerds/mentoringnetwork.

## References

Cavey, L. O., Mahavier, W. T., Parker, G. E., \& White, A. (in press) MathNerds and Mathematical Knowledge for Teaching, The Constructivist, The Association of Constructivist Teaching.
Dawkins P., De Angelis V., Mahavier W., Stenger A. (2002). MathNerds Offers Discovery-Style Mathematics on the Web, MAA Focus, 22( 2), 10-11.

## Featured Mathematics Education Article:

 Case Studies of Mathematics Teachers' Learning in an Online Study Group Randall E. Groth, Salisbury UniversityAbstract: Two teachers participating in an online study group provided the foci for in-depth case studies. Transcripts of conversations they had with colleagues about issues related to reform-oriented pedagogy were analyzed from both acquisition and participation perspectives on learning. Both teachers exhibited mainly marginal changes to their pedagogical reasoning structures and were generally resistant to adopting ideas posed during online debates. At the same time, the text-based environment provided a setting for both participants to structure their emerging thoughts about changes to their existing pedagogical reasoning structures. It also served as a forum for them to identify gaps in their personal knowledge and to obtain further professional development to address them. The methodology and theoretical perspective employed in the report provide a foundation for further research on teachers' learning in online environments.

## AMTE Connections Seeks Submissions

AMTE's newsletter, Connections, is broadening the types of submissions we want to publish. Several new columns debut in this issue, and more will appear this fall. Each new column is described below. All submissions are reviewed by the Editorial Board for quality and appropriateness for AMTE members.

## Career Column

In this column, we will solicit submissions from experienced AMTE members about finding balance within the work of teacher education. Possible topics include finding time to write and do research; connecting service, scholarship, and teaching; strategies for working efficiently as a mathematics teacher educator (MTE); navigating the career path through promotion and tenure; tips for obtaining and administering grants; and the MTE career after promotion and tenure. Submissions of 1500-2000 words should be directed to Connections editor Lynn Stallings (lstalling@kennesaw.edu).

## Favorite Problem Column

Favorite problems will be solicited from AMTE members, along with solutions and a short commentary on how they use these problems (e.g., for which courses or audiences, what tools are provided, etc.). Submissions of 1000-1200 words should be directed to column editor Libby Knott, (knott@mso.umt.edu).

## Project News

For the Project News Column, AMTE members are asked to submit descriptions of projects or centers. The purpose of the column is to inform members about projects before they get to presentation and publication stages. Submissions of 1000-1200 words should be directed to column editor, Laurie Cavey (caveylo@jmu.edu).

## Spotlight on Practice

For this column, AMTE members are asked to submit descriptions of innovative practices within the field of mathematics teacher education. Topics may include assignments or activities from mathematics or mathematics methods course, field experiences, program assessments, program design, professional development design or activities, etc. Submissions of 1500-2000 words should be submitted to column editor Teresa Gonske (tlgonske@nwc.edu).

## Point/Counterpoint

This column will present at least two submissions with differing views on a topic of interest to mathematics teacher educators. AMTE members are encouraged to use this column for dialog about an issue. Submissions of 1000-1200 words each should be directed to column editor Kathleen Lynch-Davis (lynchrk@appstate.edu).

## Reviews of Resources

Reviews of recently published books, software, or other materials related to mathematics teacher education are solicited from AMTE members. Submissions of 1000-1200 words should be directed to column editor Troy Regis (tprb62@mizzou.edu).

## Affiliate News

Affiliates are invited to submit any news of relevance to all AMTE members. Submissions of 1000-1200 words should be directed to Connections editor Lynn Stallings (lstalling@kennesaw.edu)

Other sorts of submissions are encouraged and should be sent to Connections editor Lynn Stallings (lstalling@kennesaw.edu) for review.

## (Continued from Bush, p. 7.)

determine what they really needed. I did not look very smart in these situations, and I have since learned to be more assertive about identifying needs and priorities.
Effective service providers gather a considerable amount of information before making decisions about engaging in service. To ensure that the service obtains desired outcomes, we should: (1) make sure the timing for the service is right; (2) ensure that the service is needed; and (3) guarantee that ample resources and time are available to carry out the service in timely and effective ways. Smart service providers know the critical issues to be addressed by the service and understand the barriers to be overcome in order for the service to have impact.

In conclusion, before we look inward, outward, across, and start, we must make the most important decision-that we serve. Districts, schools, colleges, and government agencies across the country need quality service in mathematics education. The students and teachers in this country deserve the best that mathematics educators have to offer. Therefore, as mathematics educators, the most important decisions are not about when, how, and where we serve. The quote by Martin Luther King, Jr. below describes the attitude that we must take in this service-oriented profession:

Every man (woman) must decide whether he (she) will walk in the creative light of altruism or the darkness of destructive selfishness. This is the judgment. Life's persistent and most urgent question is: What are you doing for others?
-Martin Luther King, Jr.
It's all service!

AMTE Connections Summer 2007

## My Favorite Problem

Teresa Gonske, Northwestern College, Minnesota
$A, B, C$, and $D$
are midpoints of the sides of a square with side length 8. If these midpoints are connected by line segments to the vertices of the square as shown (fig. 1), a square is formed. Determine the area of the square.

In this issue of Connections, several new columns debut. Favorite problems are solicited from AMTE members, along with solutions and a short commentary on how they use these problems (e.g., for which courses or audiences, what tools are provided, etc.). Submissions of 1000-1200 words should be directed to column editor Libby Knott, (knott@mso.umt.edu).

Characteristics of a "good" problem include: 1) the problem can be posed in a variety of ways, 2 ) the problem can be solved using multiple types of strategies and representations, 3) the problem lends itself to various levels of solution making it possible for students with differing ability and background to be successful, and 4) the problem can be extended or generalized.

One of my favorite problems meets these characteristics very well and I've been able to use this problem and its extensions in various forms in both mathematics content and mathematics education (methods) courses. My first encounter with this particular problem occurred in a problem solving course taught by Dr. Richard Grassl at the University of Northern Colorado. I also have a vague recollection of a similar problem in my own $7^{\text {th }}$ grade math class with Mr. Klancher at Rice Lake Middle School (Wisconsin).

Dr. Grassl posed the problem as follows: A, B, C, and D are midpoints of the sides of a square with side length 8 . If these midpoints are connected by line segments to the vertices of the square as shown (fig. 1), a square is formed. Determine the area of the square.


My own initial solution method went something like this: Label the vertices P, Q, R, and S. The area of triangles PAS and QCR are the same,

It follows that the area $(A)$ of parallelogram $A Q C S$ is $64-2(16)=32$. The base (b) of parallelogram AQCS is AS which is also the hypotenuse of triangle PAS.

Thus
and
$A=b \cdot h \Rightarrow 32=4 \sqrt{5} \cdot h \Rightarrow h=\frac{8 \sqrt{5}}{5}$. But $h$, the height of the parallelogram, is also the side of the square in the middle! So the area of the square is

$$
A=\left(\frac{8 \sqrt{5}}{5}\right)^{2}=12 \frac{4}{5}
$$

I present this particular solution, though it is not my favorite, because I find it interesting that from among all the many diverse and creative approaches my own students have used, this particular one has never been presented. This provides a good reminder that students may conceptualize the structure of a mathematical problem and formulate its solution very differently from the way we might. It is crucial to provide our preservice teachers with learning experiences that will help them realize this about their own future students.

When posing the problem to my students, I prefer to present it in a more general form without any specific measure assigned to the length of the side of the square and without indicating that the figure formed in the interior is a square. This leads to a much richer range of solution strategies because the students are not funneled into numerical thinking.

When students make the assumption that the figure in the interior is a square, I challenge that assumption and ask them prove it. This is not an easy task for them and has led to lively class discussions as they try to recall and appropriately apply properties and theorems about congruence and similar triangles, parallelograms, parallel lines, angle relationships, and slopes. Some students who are more comfortable with applying algebra than geometry will construct the object on a coordinate grid and use analytic geometry to determine slopes of the lines, write equations, find coordinates of the intersections and prove the interior figure a square from perpendicular slopes and application of the distance formula. Other students may attempt to apply trigonometry to the process.

When I use this problem in a geometry course it gives me insight into students' content knowledge and their perceptions of proof and problem solving.

On the first day of class, I have actually begun by verbally posing this version of the problem:

Construct (or draw) a square. Find the midpoint of each side. Using line segments, connect each midpoint to the two opposite vertices. Shade the interior of the polygon that is formed by the lines. Find the area of the shaded region. (fig.2)


This is a very challenging problem. (Try it yourself!) After the students work at the task for awhile and realize it is not nearly as simple as it initially seemed, I remind them that a common strategy in mathematics is to examine a simpler problem and see if solving the simpler problem gives insights. Of course the "simpler problem" is exactly the problem described previously where instead of connecting each midpoint to both of the opposite vertices we make the problem simpler by connecting each midpoint to only one of the opposite vertices.

I have asked my geometry students to keep working on this challenging problem. One of the textbooks (Kay, 2001) I've used contains a number of exercises spread throughout the text that are structurally related to this problem with the potential to help formulate a solution if the students pay close attention. The surprise that has awaited my students at the end of the course is the return of this problem as the last question on the take-home portion of the final exam. Thus the problem bookends the course!

It is common for students to initially assume, without the need for proof, that the shaded region is a regular octagon and thus the emphasis on proof with the simpler problem is helpful in this respect. Some students have used Geometer's Sketchpad ${ }^{\text {TM }}$ to construct the figure and get an initial answer by measurement. Approximately half the geometry students in a class have presented a correct solution on the final exam.

The close of the geometry course is not the last time that my mathematics education students will encounter the problem. When these students are subsequently in their secondary math methods
class, I have them read the chapter "Mathematical reasoning: In the eye of the beholder" from the 1999 NCTM Yearbook. Here Peggy House presents an eye-opening variety of diverse lines of reasoning problem solvers (both students and experienced teachers) have used in approaching this problem. This is a must-read chapter, but only after you have attempted the problem for yourself! My math education students are much better able to engage in the reading and enjoy it after they have wrestled with the problem and they are eager to discover into which category their line of reasoning fits.

So, what is my favorite solution to the problem of finding the area of the square in the interior? It is the one that is so simple that it is beautiful. Simply cut it (fig. 1) apart and rearrange the pieces to form five congruent squares! Thus the area of the interior square is $1 / 5$ that of the original square.


The first time that the "rearrange" strategy occurred to one of my students on her initial attempt occurred just this past spring. As Rachel explained, "I just looked at it and visualized rotating the triangles around 180 degrees." (fig. 3)

And that is why this is my favorite problem! It can be entered at a level that my second grade nephew can clearly understand and yet it can be used to challenge the best of my college math major students. It also can be used to motivate my preservice teachers in considering the diversity of their future students' reasoning and consequently their pedagogy choices.

## References

House, P. (1999). Mathematical reasoning: In the eye of the beholder. In L.V. Stiff \& F.R. Curcio (Eds.), Developing Mathematical Reasoning in Grades K-12 ( pp.175-187). Reston, VA: National Council of Teachers of Mathematics.
Kay, D. (2001). College geometry: A discovery approach with the Geometer's Sketchpad ${ }^{\mathrm{TM}}$. Boston: Addison Wesley Longman, Inc.

## This problem can be entered at a level that my $2^{\text {nd }}$ grade nephew can clearly understand and yet it can be used to challenge the best of my college math major students.

## Upcoming Conferences

## Online at

http://www.amte.net

Membership/Renewal Forms

Position Papers
Position Listings
Resources

Forum for Members
Other Opportunities

2007
July 29-Aug. 2 Joint Statistical Meetings
July 31-Aug. 3 AMS International
August 3-5 MAA MathFest
October 11-12 NCTM Regional
October 25-26 NCTM Regional
October 25-28 PME-NA
November 15-17 SSMA
November 29-30
$\underline{2008}$
January 6-9 MAA-AMS Joint Meeting
January 24-26
March 24-28
April 9-12
Aprile-12

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